Surface modeling in PSG

As light arrives to a surface at a particular wavelength, it can be either be absorbed or scattered. Processes such as surface fluorescence or Raman will transfer some of this energy to a different wavelength, but for our treatment in PSG, we simply consider this as an absorption process at this wavelength. The direction and intensity of the scattered light requires of complex modeling, and several methods exist (e.g., Lambert, Hapke). The light absorbed will heat the surface, and this together with other internal sources of heat will lead to thermal emission (with an associated directionality and effectiveness/emissivity). How effective the surface scatters light is defined by the single scattering albedo w, where 0 means the light is totally absorbed and to 1 the light is totally scattered back.

What is being observed or "reflected" back will depend on how this surface scatters back, and we would then require information about the observing geometry, the emissions direction and the geometry of the incidence fluxes. Three angles are used to define the geometry: i "incidence angle" is the angle between the Sun (or host-star) and the line perpendicular to the surface at the point of incidence, called the normal; e "emission angle" is the angle between the surface normal and the observer; and g "phase angle", which is the angle between the source and observer (not to be confused with solar azimuth angle, which is the projection of the phase angle).

The quantity that captures how much light is being reflected towards the observer is called r(i,e,g) "bidirectional reflectance" (r(i,e,g)=I(i,e,g)/J, where I is the scattered radiance and J is the incidence radiance) which is in units of [sr^{-1}], with steradians [sr] being a unit of solid angle. A common alternative quantity is the BRDF or "bidirectional-reflectance distribution function" [sr^{-1}], which describes the reflectivity of the surface with respect to a Lambertian sphere, and it is simply r/cos(i). Similarly for emission, directional emissivity ε(e) is the ratio of the thermal radiance emerging at emission angle e from the surface with temperature T with respect to a black body at the same temperature.

Once the geometry (i,e,g) and the specific scattering properties (e.g., w) are defined, we would then need a scattering model to accurately model the emissions from the target's surface. In PSG, four core models are available: Lambert (isotropic scattering), Hapke (parametric surface scattering), Lommel-Seeliger (weakly scattering / diffuse surfaces) and Cox-Munk (specular glint scattering model).

Lambert model: *isotropic scattering*

A Lambertian surface is one that scatters isotropically, as an ideal matte or a perfectly diffusive reflecting surface. The emissivity can be defined as ε = 1-w, while the bidirectional-reflectance distribution function is defined as (adapted from Hapke, 2012a [H12 hereafter] eq. 8.12):

$$\mathrm{BRDF} = \frac{w}{\pi}$$

Hapke model: *parametric surface modeling*

The Hapke scattering model is a physically motivated model that approximates the solution for radiative transfer for a porous, irregular and particulate surface. This model has been advancing over the last decades and captures processes and radiative transfer phenomena parameterized with approximations, which are motivated by the basic physical principles of scattering. In PSG, we implement the generic Hapke's "Isotropic Multiple-Scattering Approximation" (IMSA) model, which is useful if the surface scattering function is not too anisotropic and can be mostly described by a single-scattering term. The implementation also includes Hapke's shadow-hiding opposition effect (SHOE) factor, the coherent backscatter opposition effect (CBOE) and a compensation for surface roughness. The BRDF can be then modeled following H12 (eq. 12.55) as:

$$\mathrm{BRDF} = \mathrm{K}\frac{w}{4\pi}\frac{1}{\mu_{0e}+\mu_{e}}\left[\mathrm{P(g)}[1+\mathrm{B_{SO}B_{S}(g)}]+\mathrm{H}\left(\frac{\mu_{0e}}{\mathrm{K}}\right)\mathrm{H}\left(\frac{\mu_{e}}{\mathrm{K}}\right)-1\right][1+\mathrm{B_{CO}B_{C}(g)}]\mathrm{S(i,e,g)}$$

where K is the porosity coefficient, μ_{0e} and μ_e are the cosine of the effective incidence and emission angles respectively (see below), P(g) the phase function, B_{SO} is the amplitude of the opposition effect (0 to 1), $B_S(g)$ is the shadow-hiding opposition function, B_{CO} is the amplitude of the coherent backscatter opposition effect (0 to 1), $B_C(g)$ is the backscatter angular function, H(x) is the Ambartsumian–Chandrasekhar H function, and S is the shadowing/roughness function. The phase function can be characterized using different representations, and in PSG four functions are available: HG1, HG2, HGH and LP2.

<u>**The single lobe Henyey-Greenstein (HG1):**</u> has one parameter ξ (asymmetry parameter, -1:backscatterer to 1:forward) defined as (H12 eq. 6.5):

$$\mathrm{P_{HG1}(g)} = \frac{1-\xi^2}{\left(1+2\xi\cos\mathrm{g}+\xi^2\right)^{3/2}}$$

The sign of ξ may differ depending on the definition of the "phase" g angle and the sign of the cosine term used for the HG1 function. In PSG, g=0 implies the backward direction, and therefore negative ξ numbers imply backscattering (typical).

<u>**The double-lobed Henyey-Greenstein (HG2):**</u> has parameters b (asymmetry parameter, 0 to 1) and c (back-scattering fraction, -1 to 1) and it is defined as (H12 eq. 6.7a):

$$\mathrm{P_{HG2}(g)} = \frac{1+c}{2}\frac{1-b^2}{(1-2b\cos g+b^2)^{3/2}}+\frac{1-c}{2}\frac{1-b^2}{(1+2b\cos g+b^2)^{3/2}}$$

There are conflicting definitions of the c parameter (some use [1-c] and [c] as scalers), so please take this into account when entering this parameter into PSG.

Henyey-Greenstein Hapke/hockey phase function (HGH): It has been observed that for the HG2 function, there is an inverse relationship between the b and c parameters following a hockey stick shape. As such, the HGH phase function can be defined following Hapke, 2012b (eq. 8) as:

$$c = 3.29\exp(-17.4b^2) - 0.908$$

The two-term Legendre polynomial function (LP2) has parameters b and c, and it is defined as (H12 eq. 6.3 with P_0, P_1, P_2 defined in appendix C.4):

$$P_{LP2}(g) = 1 + b\cos g + c(1.5\cos^2 g - 0.5)$$

The shadow-hiding opposition function can be approximated following H12 (eq. 9.22) as:

$$B_S(g) = \frac{1}{1 + (1/h_S)\tan g/2}$$

where h_S is the width of the opposition surge. The backscatter angular function can be approximated following H12 (eq. 9.43) as:

$$B_C(g) = \left\{1 + [1.3 + K]\left[\left(\frac{1}{h_C}\tan\frac{g}{2}\right) + \left(\frac{1}{h_C}\tan\frac{g}{2}\right)^2\right]\right\}^{-1}$$

where h_C is the width of the backscatter function. The Ambartsumian–Chandrasekhar H function can be approximated with errors of less of than 1% and following H12 (eq. 8.56) as:

$$H(x) = \left\{1 - wx\left[r_0 + \frac{1 - 2r_0 x}{2}\ln\left(\frac{1+x}{x}\right)\right]\right\}^{-1}$$

where r_0 is the diffusive reflectance, which is calculated from the albedo factor $\gamma = (1-w)^{1/2}$ as (H12 eq. 8.25):

$$r_0 = \frac{1-\gamma}{1+\gamma}$$

The porosity coefficient K is dependent on ϕ, the filling factor or fractional volume filled by material (0: loose grains, 1: highly compacted material), given by (H12 eq. 7.45b):

$$K = \frac{-\ln\left(1 - 1.209\phi^{2/3}\right)}{1.209\phi^{2/3}}$$

When employing the roughness term S, this implementation impacts the effective cosine of the incidence angles ($\mu_0 \rightarrow \mu_{0e}$) and emission angles ($\mu \rightarrow \mu_e$), where μ_0=cos(i) and μ=cos(e). The shadowing term and the new μ_{0e} and μ_e are calculated following H12 (eq. 12.63) as:

when i ≤ e:

$$\mu_{0e} = \chi(\theta_P)\left[\cos i + \sin i \tan\theta_p \frac{\cos\psi\, E_2(e) + \sin^2(\psi/2)\, E_2(i)}{2 - E_1(e) - (\psi/\pi)E_1(i)}\right]$$

$$\mu_e = \chi(\theta_P)\left[\cos e + \sin e \tan\theta_p \frac{E_2(e) - \sin^2(\psi/2)\, E_2(i)}{2 - E_1(e) - (\psi/\pi)E_1(i)}\right]$$

$$S = \frac{\mu_e}{\eta(e)}\frac{\mu_0}{n(i)}\frac{\chi(\theta_p)}{1 - f(\psi) + f(\psi)\chi(\theta_p)[\mu_0/\eta(i)]}$$

when e ≤ i:

$$\mu_{0e} = \chi(\theta_P)\left[\cos i + \sin i \tan\theta_p \frac{E_2(i) - \sin^2(\psi/2)\, E_2(e)}{2 - E_1(i) - (\psi/\pi)E_1(e)}\right]$$

$$\mu_e = \chi(\theta_P)\left[\cos e + \sin e \tan\theta_p \frac{\cos\psi\, E_2(i) + \sin^2(\psi/2)\, E_2(e)}{2 - E_1(i) - (\psi/\pi)E_1(e)}\right]$$

$$S = \frac{\mu_e}{\eta(e)}\frac{\mu_0}{n(i)}\frac{\chi(\theta_p)}{1 - f(\psi) + f(\psi)\chi(\theta_p)[\mu/\eta(e)]}$$

where

$$\theta_p = (1 - r_0)\theta$$

$$\psi = \mathrm{acos}\left(\frac{\cos g - \cos i \cos e}{\sin i \sin e}\right)$$

$$f(\psi) = \exp\left(-2\tan(\psi/2)\right)$$

$$\chi(\theta_p) = (1 + \pi\tan{\theta_p}^2)^{-1/2}$$

$$E_1(y) = \exp\left(-2/\pi \cot\theta_p \cot y\right)$$

$$E_2(y) = \exp\left(-1/\pi \cot^2\theta_p \cot^2 y\right)$$

$$\eta(y) = \chi(\theta_p)\left[\cos y + \sin y \tan\theta_p (E_2(y)/(2 - E_1(y))\right]$$

Thermal emission from a scattering surface will also have directionality, and the directional emissivity of an optically thick particulate medium can be defined following H12 (eq. 15.19) as:

$$\varepsilon = \gamma H(\mu)$$

The table below summarizes the parameters needed by PSG when performing Hapke modeling and their typical range. For comparison Hapke parameterizations and derivations for objects across our solar system are also listed, in which P2020: (Protopapa et al., 2020), B2020: (Belgacem et al., 2020), F2015: (Fernando et al., 2015), S2014: (Sato et al., 2014), HV1989: (Helfenstein and Veverka, 1989), and L2015: (Li et al., 2015).

Hapke parameter	**Range**	**Hapke parameters for objects in our solar system**					
		Pluto P2020	Europa B2020	Mars F2015	Moon S2014	C-type asteroids HV1989	S-type asteroids HV1989
P(g) phase function	HG1, HG2, HGH, LP2	HG1	HG2	HG2	HGH	HG1	HG1
ξ or b phase coefficient	-1 to 1	-0.36	0.2 to 0.6	0.2 to 0.6	0.1 to 0.3	-0.47	-0.27
c phase coefficient	-1 to 1	-	0.1 to 0.9 alternative	0.1 to 1.0	-	-	-
B_{SO} opposition surge scaler	0 to 1	0.307	0.2 to 0.9		1.5 to 2.1	1.03	1.6
h_S opposition surge width	≥ 0	0.206	0.2 to 0.7		0 to 0.12	0.025	0.08
θ roughness mean slope angle [degree]	0 to 40	20	6 to 27	5 to 25	23.4	20	20
ϕ filling factor	0 to 0.75	0.0	0.0	0.0	0.0	0.0	0.0
B_{CO} coherent backscattering scaler	0 to 1	0.074	0.0	0.0	0.0	0.0	0.0
h_C width of coherent backscattering	≥ 0	0.0017	0.0	0.0	0.0	0.0	0.0

Lommel-Seeliger: *dark and weakly scattering Lunar/aerosol surfaces*

For relatively dark objects with weakly scattering surfaces, the Lommel-Seeliger model performs well in capturing the variation of the scattered fluxes with respect to the source/observational angles. It is therefore the preferred model when interpreting the Moon, asteroids and other small bodies. The generalized Lommel-Seeliger is defined as (adapted from H12 eq. 8.35a):

$$\mathrm{BRDF} = \frac{w}{4\pi}\frac{1}{\mu + \mu_o}\mathrm{P(g)}$$

where w is the surface single scattering albedo and P(g) is the single-scattering phase function. Several disk-resolved models are based on this basic formalism (e.g., ROLO), and since this model is suitable for small unresolved dark bodies, it is the preferred method in PSG for modeling the disk-resolved BRDF of asteroids and comets. In the literature, there are several measurements and derivations of the "phase function" for unresolved bodies, but these refer to the integral phase function Φ(g), not to P(g). We can re-normalize the empirically derived Φ(g) to an effective surface P(g) by dividing by the integrated reflectance of a perfect Lommel-Seeliger object (adapted from H12 eq. 6.14):

$$\frac{\Phi(\mathrm{g})}{\mathrm{P(g)}} = \left[1 - \sin\frac{\mathrm{g}}{2}\tan\frac{\mathrm{g}}{2}\ln\left(\cot\frac{\mathrm{g}}{4}\right)\right]$$

<u>**Lumme-Bowell phase function (HG):**</u> The Lumme-Bowell model is a scattering model typically used in asteroid research and presented in Lumme and Bowell (1981). A simplified empirical version of the integral Lumme–Bowell model was adopted by the International Astronomical Union (IAU) in 1985 to describe the integral phase function of asteroids, and this function with slope parameter is adopted as (H12 section 12.5.2):

$$\Phi(\mathrm{g}) = (1 - \mathrm{G})\Phi_1(\mathrm{g}) + \mathrm{G}\Phi_2(\mathrm{g})$$

$$\Phi_1(\mathrm{g}) = \exp[-3.33(\tan \mathrm{g}/2)^{0.63}]$$

$$\Phi_2(\mathrm{g}) = \exp[-1.87(\tan \mathrm{g}/2)^{1.22}]$$

where Φ_1(g) is the single scattering component (steep function, ~0.043 mag/deg), Φ_2(g) the multiply scattered component (shallower, ~0.014 mag/deg), and G is the slope parameter ($0 \leq G \leq 1$). Considering that the geometric albedo (A_{geo}) of a LS object is w/8, the single scattering albedo w can be determined from A_{geo} and the disk-integrated absolute magnitude H_0 value and the object's diameter (D) as (Tedesco et al., 1992):

$$\mathrm{A_{geo}} = \frac{w}{8} = \left(\frac{1329}{\mathrm{D[km]}}10^{-0.2\mathrm{H_0}}\right)^2$$

Muinonen 3-parameters (HG_1G_2): over the last decades, it was observed that several bodies could not be properly described with the HG phase function, and a new system with three parameters was developed (Muinonen et al., 2010). In 2012, the IAU replaced the HG system with the HG_1G_2 system. The integral phase function, with splines coefficients listed in (Penttilä et al., 2016), is described as:

$$\Phi(g) = G_1\Phi_1(g) + G_2\Phi_2(g) + (1 - G_1 - G_2)\Phi_3(g)$$

$$\text{Splines of } \Phi_1(g) = 1 - 6g/\pi$$

$$\text{Splines of } \Phi_2(g) = 1 - 9g/(5\pi)$$

$$\text{Splines of } \Phi_3(g) = \exp\left(-4\pi \tan^{2/3} g/2\right)$$

Pentilla 2-parameters (HG_{12}): Penttilä et al. (2016) determined an improved relationship between the G1 and G2 parameters, which is applicable to all types of asteroids with the exception of E- and D-types:

$$G_1 = 0.5351335 \cdot G_{12}$$

$$G_2 = 0.84293649 - 0.5351335 \cdot G_{12}$$

where G_{12} is only valid between 0 and 1.

Exponential (EXP): an exponential empirical series was investigated for the OSIRIS-REx mission to asteroid Bennu (Takir et al., 2015) as:

$$P(g) = \exp\left(\beta g + \gamma g^2 + \delta g^3\right)$$

Lunar/ROLO: The ROLO model was developed by (Buratti et al., 2011), using the USGS's ROLO data from NASA's Moon Mineralogy Mapper (M3), and the surface phase function and single scattering albedo can be described following (Buratti et al., 2011) as:

$$P(g) = C_0 \exp(-C_1 g) + A_1 g + A_2 g^2 + A_3 g^3 + A_4 g^4$$

$$w = 4(A_0 - C_0) = 8\, A_{geo}$$

The table below summarizes the parameters needed by PSG when employing the LS model and their typical value ranges. For comparison, parameterizations and derivations for objects across our solar system are also listed, in which T2015: (Takir et al., 2015), C2017: (Ciarniello et al., 2017), B2011:(Buratti et al., 2011), V2015: (Vereš et al., 2015).

LS model parameter	Range	Parameters for objects in our solar system					
		Bennu T2015	67P C2017	Ceres C2017	Moon B2011	C-type asteroids V2015	S-type asteroids V2015
Φ(g) Phase function	HG, HG1G2, HG12, EXP, ROLO	EXP	HG	HG	ROLO	HG12	HG12
G, a_1, G_{12}, β, C_0	See text	-0.043	-0.09	0.02	0.2-0.4 10^{-2}	0.58	0.47
a_2, γ, C_1		2.6 10^{-4}			0.04 to 0.23		
a_3, δ, A_1		-9.7 10^{-7}			-0.6 to 0.1 10^{-2}		
A_2					-1 to 1 10^{-4}		
A_3					-4 to 2 10^{-6}		
A_4					-1 to 2 10^{-8}		

Cox-Munk model: *glint and ocean's reflections*

The Cox-Munk model is a scattering model of glitter on a water surface. The model employs geometric optics model with the assumption of a Gaussian distribution of the slopes of the wave facets. In the implementation of the glint model in PSG, the BRDF includes two terms, the pure glint term (Cox and Munk, 1954; Jackson and Alpers, 2010; Ma et al., 2015; Spurr, 2002), and the classical non-glint Lambert term for the surface:

$$\mathrm{BRDF} = \mathrm{BRDF}_{\mathrm{glint}} + \mathrm{BRDF}_{\mathrm{Lambert}}$$

$$\mathrm{BRDF}_{\mathrm{glint}} = \frac{\mathrm{r\ \ p\ \ s_{\Lambda}\ \ (1 + tan^2\ \beta)^2}}{4\cos \mathrm{e}}$$

where r is the Fresnel reflection coefficient for an unpolarized source and computed as:

$$\mathrm{g_r} = \mathrm{asin}(\sin \mathrm{g}\,/1.34)$$

$$\mathrm{r} = \frac{1}{2}\left[\left(\frac{\sin(\mathrm{g}-\mathrm{g_r})}{\sin(\mathrm{g}+\mathrm{g_r})}\right)^2 + \left(\frac{\tan(\mathrm{g}-\mathrm{g_r})}{\tan(\mathrm{g}+\mathrm{g_r})}\right)^2\right]$$

Cox and Munk (1954) found that the probability density function of the wave slopes depends on the wind speed (U_{wind}, assumed 4 m/s when not provided), and the probability determining glint reflections can be approximated by a Gaussian function as:

$$\sigma^2 = 0.003 + 0.00512\ \mathrm{U_{wind}}[\mathrm{m/s}]$$

$$\cos\beta = \frac{\cos i + \cos e}{2\cos g}$$

$$p = \frac{1}{\pi\sigma^2}\exp\left(-\frac{\tan^2\beta}{\sigma^2}\right)$$

As we approach high incidence angles, not all facets are visible, and the "shadow" term compensates for this:

$$s_\Lambda = \frac{1}{1 + \Lambda(i) + \Lambda(e)}$$

$$\Lambda(x) = \frac{1}{2}\left\{\frac{1}{\sqrt{\pi}}\frac{\sigma}{\cot\ x}\exp\left(-\frac{\cot^2 x}{\sigma^2}\right) - \operatorname{erfc}\left(\frac{\cot x}{\sigma}\right)\right\}$$

where erfc(x) is the complementary error function.

Cometary dust/icy grains

Dust and icy particles in cometary comae are in many cases the main source of continuum/broad radiation in small bodies and in exospheres. In PSG, the single scattering albedo (*w*) of the nucleus and the dust grains are assumed to be the same, yet their scattering properties are treated differently. Small particles have a very different response to the solid nuclear body, with a strong forward scattering peak and a less shallow scattering phase function. PSG employs a Lambert model with the Halley-Marcus (H-M) integral phase function compiled by (Schleicher and Bair, 2011) to model the scattering properties for the dust grains, independently of the selected nucleus scattering/phase model.

The intensity of the continuum would then depend on the effective emitting area of the dust grains, and for that we employ a model as described in (Villanueva et al., 2018), which is dependent on the comet's activity and has been scaled to match an empirical relationship of cometary brightness and cometary activity (Jorda et al., 2008). The user can use this model and the A(Θ)fρ method to determine the continuum intensity:

Dust/gas ratio: in this approach the dust particles are treated as behaving like the surrounding gas and a dust/gas mass ratio of 1.0 provides consistent continuum fluxes to the brightness vs. gas-activity relationship. The reflected sunlight flux is affected by the H-M phase curve, while the thermal emission is assumed to be isotropic and not affected by phase.

A(Θ)fρ: is a quantity introduced by (A'Hearn et al., 1984) that describes continuum intensity and is generally independent of the different image-scale and measuring window sizes used in the photometry. Since this quantity intrinsically includes a phase correction, the A(Θ)fρ reflected fluxes are not corrected by the H-M phase curve, yet the thermal fluxes are corrected by $1/P_{HM}(\Theta)$.

Mixing compositions

In PSG, mixing of components is done via the "areal" mixture principle. In an areal mixture, the surface area viewed by the detector consists of several unresolved, smaller patches, each of which consists of a pure material. In this case the total reflectance is simply the linear sum of each reflectance weighted by area. If these components abundances total than unity, then the total surface reflectance ($BRDF_T$) and emissivity (ε_T) is complemented by the entered generic surface "albedo" a_0 and "emissivity" ε_0 as (adapted from H12 eq. 10.42):

$$F_T = \sum_j F_j$$

$$BRDF_T = \sum_j F_j BRDF(w_j) + (1 - F_T) BRDF(a_0)$$

$$\varepsilon_T = \sum_j F_j \varepsilon(\varepsilon_j) + (1 - F_T)\varepsilon(\varepsilon_0)$$

where F_j is the fractional area of this component with respect to the total sampled scene.

Calculation of the single scattering albedo: *reflectances, optical constants and albedos*

For each of the models described above, a key parameter is the single scattering albedo (w). This parameter can be calculated for a specific surface from optical constants, or it can be determined from laboratory measurements of reflectance of that component, or it can be derived from astronomical measurements (e.g., geometric albedo). Scattering albedo, geometric albedo, Bond albedo, reflectance, absorptivity are all related quantities, yet they have very different meanings and their values can differ greatly for the same component. For instance, how can one use a "reflectance" laboratory spectrum with the models previously described? One would need to convert these to a wavelength dependent single scattering albedo (w), and for that we would need the exact sample properties (e.g., compactness) and observing conditions as employed in the laboratory experiment.

Reflectances: if the user provides an average "albedo" or "reflectance" (R) or employs reflectance databases (e.g., USGS), PSG will scale these to derive the representative single scattering albedo (w), depending on the selected scattering model. For the Lambert model (and Cox-Munk Lambert component) and the Lommel-Seeliger model, w is simply assumed to be R. For the Hapke model, the single scattering albedo (w) is calculated assuming that the laboratory/input reflectance R defines Hapke's diffusive reflectance parameter (r_0), and therefore:

$$w_{Hapke} = \frac{4R}{(1+R)^2} \qquad R = \frac{1 - \sqrt{1 - w_{Hapke}}}{1 + \sqrt{1 - w_{Hapke}}}$$

Alpha parameter (attenuation coefficient): for species described with an "attenuation coefficient" (α), the single scattering albedo is calculated as $w = \exp(-\alpha h)$, where h is the thickness (or mean ray path length) of the material on the surface.

Optical constants: when optical constants (n and k) are provided, the single scattering albedo (w) at wavelength λ for a slab of thickness h is calculated following H12 (section 6.5.3, w from eq. 6.20, S_e from eq. 5.37, S_i from eq. 6.23, θ from eqs. 5.56 and 5.8):

$$w = S_e + (1 - S_e)\frac{1 - S_i}{1 - S_i\Theta}\Theta$$

$$S_e = 0.0587 + 0.8543\,\Gamma + 0.0870\,\Gamma^2$$

$$S_i = 1 - \frac{1}{n^2}[0.9413 - 0.8543\,\Gamma - 0.087\,\Gamma^2]$$

$$\Gamma = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2}$$

$$\Theta = \exp\left[-4\pi k\frac{h[\mu m]}{\lambda[\mu m]}\right]$$

Geometric albedo (A_{geo}) or physical albedo: this is an apparent quantity that specifies how bright the whole planet/object appears for its size (idealized flat disk) at phase=0 (as seen from the Sun/star). A_{geo}=1 means that all the light arriving is reflected back, and A_{geo} can also be greater than 1 if the object has a strong opposition effect. Considering that a planetary disk encounters the full range of incidence / emission angles, the relationship between A_{geo} and w will differ depending on the assumed surface scattering model (Shepard, 2017, H12 eq. 11.34):

$$A_{geo}^{Lambert} = \frac{2}{3}w = \frac{2}{3}R \qquad A_{geo}^{LS} = \frac{1}{8}wP(0) = R\,P(0)$$

$$A_{geo}^{Hapke} = \left\{\frac{w}{8}[P(0)(1 + B_{s0}) - 1] + (0.49\,r_0 + 0.19\,r_0^2)\right\}(1 + B_{S0})$$

Bond albedo (A_{Bond}): this quantity defines how much radiation the surface scatters across all wavelengths and all directions. The Bond albedo is a value strictly between 0 and 1, as it includes all possible scattered light (but not radiation from the body itself). Bond albedo is particularly relevant when investigating the energy balance of a planet, yet it should not be used when predicting the brightness of an object at a certain wavelength, since this quantity effectively describes the average response across all wavelengths.

Emissivity (ε): this quantity defines the efficiency of a surface in radiating its thermal energy. Considering energy conservation and Kirchhoff's law, the emissivity could be defined to be equal to

1 minus the absorptivity when integrating across all wavelengths, yet emissivity could exceed unity at certain wavelengths and directions. Absorptivity and Bond albedo are closely related, but not exactly the same, and in many cases the relationship 1-albedo can be assumed. In a general case, emissivity can have "direction" and specific response at a certain wavelength, and as reported above, for each scattering we define a method to compute emissivity from the scattering albedo.

Disk integrated quantities: albedos and phase integrals

One important aspect of the BRDF quantity is that it refers to a spatially defined location on the planet's surface, with a specific bi-directionality between the source (i angle) and the observer (e and g angles). In many cases, the observer's field-of-view (FOV) may encompass a broad range of incidence and emission angles, as when we measure the spectra of unresolved small-bodies. We would then need the integral of the bi-directional reflectance across the sampled region, or disk-integrated reflectance when the whole hemisphere is sampled. One important quantity is then the integral phase function Φ(g), which defines how the brightness of the planet/object changes when observed at different phases with respect to opposition (g=0). As we discussed above, at phase g=0, A_{geo} defines the average reflectivity at opposition, while Φ(g) operates as a scaling factor for other phase angles and normalized to 1.0 for g=0. The phase integral is defined for each scattering modeling as following (Shepard, 2017, H12 eq 11.42):

$$\Phi(\mathrm{g})_{Lambert} = \frac{1}{\pi}(\sin \mathrm{g} + (\pi - \mathrm{g}) \cos \mathrm{g})$$

$$\Phi(\mathrm{g})_{LS} = \mathrm{P}(\mathrm{g}) \left[1 - \sin\frac{\mathrm{g}}{2}\tan\frac{\mathrm{g}}{2}\ln\left(\cot\frac{\mathrm{g}}{4}\right)\right]$$

$$\Phi(\mathrm{g})_{Hapke} = \frac{r_0}{2A_{geo}}\left\{\left[\frac{(1+\gamma)^2}{4}\{[1 + B_{S0}B_S(\mathrm{g})]\mathrm{P}(\mathrm{g}) - 1\} + [1 - r_0]\right]\right.$$
$$\times \left[1 - \sin\frac{\mathrm{g}}{2}\tan\frac{\mathrm{g}}{2}\ln\left(\cot\frac{\mathrm{g}}{4}\right)\right]$$
$$\left. + \frac{4}{3}r_0\frac{\sin \mathrm{g} + (\pi - \mathrm{g}) \cos \mathrm{g}}{\pi}\right\}[1 + B_{C0}B_C(g)]$$

These integral formalisms are only provided for reference, since PSG performs the integrals across the field-of-view numerically. Specifically, the geometry module in PSG computes a scaling factor to the discrete reflectances at the i,e,g employed by the radiative transfer module, with respect to the integrated reflectance when diverse angles encompassed by the FOV are considered. This integration is performed numerically for the average w by dividing the disk in 140 x 140 pixels (19600 pixels), and a scaling factor between the FOV/disk-integrated and disk-resolved scattering model is determined. This allows PSG to compute accurately the radiating fluxes even when the FOV encompasses a large fraction of the disk and is offset from the object center.

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